

Practice Algebra Exam – Math 1060Q

This practice exam is longer than the actual exam will be. The actual exam will be 10 questions.

1. Solve for x : $x^2 - 6x = 27$.

Solution:

$$\begin{aligned} x^2 - 6x &= 27 \\ x^2 - 6x - 27 &= 0 && \text{subtract 27 from both sides} \\ (x - 9)(x + 3) &= 0 && \text{factor} \end{aligned}$$

Now set each factor equal to 0 separately to get the solution:

$$\{9, -3\}$$

2. Solve for y : $\frac{3 + 5y}{y} = 2y$

Solution:

$$\begin{aligned} \frac{3 + 5y}{y} &= 2y \\ 3 + 5y &= 2y^2 && \text{multiply by } y \text{ to clear denominator} \\ 2y^2 - 5y - 3 &= 0 && \text{get all terms on the same side} \end{aligned}$$

Now we need to factor if we can, or we can use the Quadratic Formula to find the solutions. Let's use the Quadratic Formula here.

$$\begin{aligned} x &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)} \\ &= \frac{5 \pm \sqrt{25 + 24}}{4} \\ &= \frac{5 \pm 7}{4} \end{aligned}$$

We now get one solution as $\frac{5+7}{4}$ and one from $\frac{5-7}{4}$. These give us the following solutions:

$$\{3, -\frac{1}{2}\}$$

(Alternatively, we could have factored the quadratic to get $(2y + 1)(y - 3) = 0$.)

3. Simplify as much as possible: $\frac{x^{-1} + x}{\frac{1}{x}}$.

Solution:

$$\begin{aligned} \frac{x^{-1} + x}{\frac{1}{x}} &= \left(\frac{x^{-1} + x}{1} \right) \left(\frac{x}{1} \right) && \text{dividing by fraction is multiplying by reciprocal} \\ &= (x^{-1} + x)x \\ &= x^{-1}x + x^2 && \text{distribute} \\ &= x^0 + x^2 && \text{property of exponents} \\ &= 1 + x^2 \end{aligned}$$

4. State the Quadratic Formula.

Solution: You can try singing it to the tune of “Pop Goes the Weasel” if you have trouble remembering.

5. Insert parentheses in two different ways to make this statement mean two different things:
 $ab^3 + b - c$.

Solution: Many right answers but here are two possibilities.

$$(ab)^3 + b - c$$

$$a(b^3) + b - c$$

6. Factor completely: $6x^4 + 3x^3 - x^2$.

Solution: Start by pulling out common factors.

$$6x^4 + 3x^3 - x^2 = x^2(6x^2 + 3x - 1)$$

Now you must try to factor the quadratic by whatever technique you know. I will use the quadratic equation to find the zeros and get the factors from them.

$$\begin{aligned} x &= \frac{-3 \pm \sqrt{3^2 - 4(6)(-1)}}{2(6)} \\ &= \frac{-3 \pm \sqrt{9 + 24}}{12} \end{aligned}$$

Here we can notice that the square root will not be a perfect square. This tells us that the answer will be irrational, so factoring will be messy. Therefore, our answer is:

$$x^2(6x^2 + 3x - 1)$$

(If we went ahead and factored the quadratic, it would look like this: $x^2(x - \frac{-3+\sqrt{9+24}}{12})(x - \frac{-3-\sqrt{9+24}}{12})$.)

7. Factor completely: $\frac{5ux^3}{y^2} + \frac{3uxy}{4}$.

Solution: Start by getting a common denominator

$$\begin{aligned} \frac{5ux^3}{y^2} + \frac{3uxy}{4} &= \left(\frac{4}{4}\right) \left(\frac{5ux^3}{y^2}\right) + \left(\frac{3uxy}{4}\right) \left(\frac{y^2}{y^2}\right) \\ &= \frac{20ux^3 + 3uxy^3}{4y^2} && \text{combine to one fraction} \\ &= \left(\frac{ux}{1}\right) \left(\frac{20x^2 + 3y^3}{4y^2}\right) && \text{pull out common factors} \end{aligned}$$

What's left is unfactorable, so our answer is:

$$ux \left(\frac{20x^2 + 3y^3}{4y^2}\right)$$

Or, you could factor directly:

$$ux \left(\frac{5x^2}{y^2} + \frac{3y}{4}\right)$$

8. Simplify: $\frac{\frac{3y}{xz}}{\frac{z^2y}{x}}$.

Solution:

$$\begin{aligned}\frac{\frac{3y}{xz}}{\frac{z^2y}{x}} &= \left(\frac{3y}{xz}\right) \left(\frac{x}{z^2y}\right) && \text{multiply by reciprocal} \\ &= \frac{3yx}{xz^3y} \\ &= \frac{3}{z^3} && \text{cancel common factors in top and bottom}\end{aligned}$$

9. Which of the following are true? Circle all that are true.

$$3\frac{a}{b} = \frac{3a}{3b} \quad 3(a-b) - 3b = 3a \quad (ab)^3 = a^3b^3 \quad (a+b)^3 = a^3 + b^3$$

Solution:

The first is false, since laws of fraction multiplication would only multiply the 3 into the numerator: $3\frac{a}{b} = \frac{3a}{b}$.

The second is also false, since $3(a-b) - 3b = 3a - 3b - 3b = 3a - 6b$.

The third is true, it is a property of exponents. $(ab)^3 = (ab)(ab)(ab) = (aaa)(bbb) = a^3b^3$.

The fourth is false, since: $(a+b)^3 = (a+b)(a+b)(a+b) = (a^2 + 2ab + b^2)(a+b) = a^3 + 3a^2b + 3ab^2 + b^3$. You can't distribute exponents over addition!

10. Expand so that there are no parentheses: $5(x^3 + 2)^2$.

Solution:

$$\begin{aligned}5(x^3 + 2)^2 &= 5(x^3 + 2)(x^3 + 2) \\ &= 5(x^6 + 4x^3 + 4) && \text{foil and combine like terms} \\ &= 5x^6 + 20x^3 + 20 && \text{distribute the 5}\end{aligned}$$

11. Solve for x : $x^2 = x^4$.

Solution:

$$\begin{aligned}x^2 &= x^4 \\ x^4 - x^2 &= 0 && \text{collect terms on one side} \\ x^2(x^2 - 1) &= 0 && \text{factor out common terms} \\ x^2(x-1)(x+1) &= 0 && \text{factor the remaining quadratic}\end{aligned}$$

Now set each factor equal to zero to get out full solution: $\{0, -1, 1\}$.

12. Are there any errors in the following solution? If so, circle each error and explain. If not, write "No Errors."

$$\begin{aligned}\frac{4}{x} + \frac{x}{2} &= 9 && \text{(equation)} \\ \frac{4}{x} + \frac{x}{2} &= 9 && \text{(cross-multiply on left-hand side)} \\ 8 + x^2 &= 9 && \text{(subtract 8 from both sides)} \\ x^2 &= 1 && \text{(take square root of both sides)} \\ x &= 1 && \text{(final answer)}\end{aligned}$$

Solution: Cross-multiplying is a valid operation if you have two fractions on either side of an equality. Otherwise, by cross-multiplying you're changing the value of an expression, so that's a bad step. We also forgot to consider negative square roots in the last step.

13. Subtract and simplify as much as possible: $\frac{4}{x+1} - \frac{x}{1-x}$.

Solution:

$$\begin{aligned}\frac{4}{x+1} - \frac{x}{1-x} &= \frac{4(1-x) - x(x+1)}{(x+1)(1-x)} && \text{common denominator and combine} \\ &= \frac{4 - 4x - x^2 - x}{(x+1)(1-x)} && \text{distribute} \\ &= \frac{-x^2 - 5x + 4}{(x+1)(1-x)}\end{aligned}$$

To check if any of the factors in the bottom are also factors in the top, and would hence cancel, we can check if they share any zeros. The denominator clearly has zeros $\{1, -1\}$. Plugging them each into the top we see that neither is a zero, and therefore no factors are shared. Now we can foil out the bottom to get a final answer.

$$\frac{-x^2 - 5x + 4}{-x^2 + 1}$$

14. Solve for x : $\sqrt{x}(3x - 2) = 0$.

Solution: Set each of the factors equal to zero to get the solutions $\{0, \frac{2}{3}\}$

15. Find the distance between the points $(-1, -2)$ and $(3, 4)$.

Solution: $d = \sqrt{(3 - (-1))^2 + (4 - (-2))^2} = \sqrt{16 + 36} = \sqrt{52}$

16. Describe in words the set of points in the plane given by the relation $(x - 2)^2 + y^2 - 4 = 0$. Is the origin an element of this set?

Solution: This is “the set of points exactly 2 units from the point $(2,0)$ ” or, in other words, “a circle with radius 2 centered at the point $(2,0)$.” To see if the origin is in the set, we can plug $x=0$ and $y=0$ into the equation and see if it stays true. $(0 - 2)^2 + 0^2 = 4$, so yes the origin is in the set.

17. Rewrite $(-\infty, 4) \cup [3, 7]$ as a single interval, and rewrite $(-\infty, 4) \cap [3, 7]$ as a single interval.

Solution: A union means include everything in either set. so put both intervals on a number line together to see that:

$$(-\infty, 4) \cup [3, 7] = (-\infty, 7].$$

An intersection means include only things in both sets. Use a number line and look at where the two intervals overlap to see that:

$$(-\infty, 4] \cap [3, 7] = [3, 4).$$

18. Find the solution set to $x(x - 3) > 4$ and write it using interval notation.

Solution: First "solve" to make a 0 on one side and then factor like you would in an equation.

$$(x - 4)(x + 1) > 0$$

Now we see that we need a product to be positive, so either both factors are negative OR both factors are positive.

1. If both are negative, we get that $x < 4$ AND $x < -1$. "And" is an intersection, so find the overlap in these two sets. So both factors are negative when $x < -1$, or when x is in the set $(-\infty, -1)$ in interval notation.
2. If both are positive, we do the same thing and get that $x > 4$, or that $(4, \infty)$ is the set of solutions in that case.

Since we needed one of these OR the other, we take a union. Since the two are completely separate there will be no easier way to write the answer, so we get:

$$(-\infty, -1) \cup (4, \infty)$$

19. Solve for x : $2|x + 7| - 3 = -1$.

Solution: In solving the equation there will be two cases. if $x + 7$ is positive, then $|x + 7| = x + 7$, but if $x + 7$ is negative, then $|x + 7| = -(x + 7)$.

1. First, if $x + 7$ is positive:

$$\begin{aligned} 2(x + 7) - 3 &= -1 \\ 2x + 7 - 3 &= -1 \\ 2x + 4 &= -1 \\ 2x &= -5 \\ x &= \frac{-5}{2} \end{aligned}$$

We should make sure that this value indeed makes $x + 7$ positive since that's the case we are currently looking at, and since it does it will be part of the solution.

2. Now we do the same for when $x + 7$ is negative:

$$\begin{aligned} 2(-(x + 7)) - 3 &= -1 \\ 2(-x - 7) - 3 &= -1 \\ -2x - 14 - 3 &= -1 \\ -2x - 17 &= -1 \\ -2x &= 16 \\ x &= -8 \end{aligned}$$

Again we should make sure this solution makes $x + 7$ negative, which it does.

So our final solution is: $\{-8, \frac{-5}{2}\}$.

20. Fill in a value for a so that this equation has no solutions: $|x - 4| + a = 5$.

Solution: Here we should note that the absolute value can't be less than 0, so the left side will be no SMALLER than whatever we choose for a . Therefore if we let a be larger than 5, the equation can't work. $a = 6$ is one such answer.